Trace and chiral anomalies in QED and their underlying theory interpretation

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Abstract

Parametrizing the possible underlying theory or new physics' decoupling effects in the most general way we reexamined the validity of canonical trace relation and chiral symmetry in certain one-loop two-point functions. The anomalies and the relation between chiral and trace Ward identities are investigated and interpreted in the perspective of a complete underlying theory or new physics instead of regularization, with some new phenomena found as by-products.

I. INTRODUCTION

It has been a standard point of view that quantum field theories for particle physics and others are just effective theories (EFT) for describing 'low energy' physics in a less sophisticated way due to UV divergences as crucial underlying short distance information is lost [1]. However, these shortcomings can be consistently removed though the renormalization program that is rigorously established at least in perturbation theory [2]. But there arises a new problem with the renormalization that some classical or canonical symmetries[1] are violated. As chiral anomaly have direct physical and topological connections [3] and similarly for trace anomaly [4,5], such anomalies are often termed as quantum mechanical violation of classical symmetries. That is, the quantization procedure is incompatible with such symmetries.

However, in 't Hooft's proposal an alternative view of chiral anomaly is advanced, the chiral anomaly could arise from the decoupling of heavy fermions [6], hence one could alter the anomaly through adding or removing matter contents. Thus anomaly is more closely related with dynamical contents, or the quantum mechanical interpretation of anomaly is not quite concrete. In other words, anomaly could be replaced by 'something' normal in a complete underlying theory or framework, like the long-pursued Theory of Everything. But before this complete theory is firmly established and profoundly tested, we must work with the effective theories and parametrize our ignorance somehow. In most literature the underlying structures (or new physics, in most authors' terminology) are simply parametrized in a specific regularization scheme and it is tacitly assumed that the conclusions are regularization scheme independent. However, such simplified and regularization-based analysis might not be quite pertinent or trustworthy, sometimes even misleading [7]. In our opinion, one should seek as general as possible parametrization for the underlying or new physics in order to avoid the limitations associated with a specific regularization. We emphasize that it is the *necessity* of introducing of regularization but not the form of a regularization imply that there must be underlying structures and we must parametrize their existence properly.

In this report, we follow and elaborate on the EFT philosophy (and the principle implied in 't Hooft's proposal) to reexamine the chiral and trace Ward identities in the two point functions involving two axial current(scalar)s. We will calculate the one loop amplitudes in two approaches: (1) dimensional regularization; (2) a general differential equation approach. The rationality for the latter will be given in next section. Here we mention that this is to try to parametrize all 'possible' underlying theory's influences.

This paper is organized in the following way: In Sec. II we review the general considerations concerning the EFT philosophy and the rationale for the differential equation approach for computing the loop diagrams. In Sec. III the approach is applied to the two and three point functions. In Sec. IV we discuss the trace and chiral Ward identities parametrized by the method introduced in Sec. II. Then in Sec. V we propose a normal interpretation of any anomalies. The paper is summarized in Sec. VI.

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[1]Here and in the following, the symmetry refers to any classical relations. For example, there could be mass term that softly violate the exact chiral and scale symmetries

II. GENERAL CONSIDERATIONS FROM UNDERLYING THEORY SCENARIO

The EFT philosophy should carry at least two different technical aspects: (1) the interaction and matter content are not complete and (2) the theory's quantum structures are not well defined in the full spectrum. In principle one could imagine that the second is just a special case of the first one in some sense. But no matter what kind of underlying theory(new physics) are to be found, there must be some missing parameters or constants that are necessary to make the EFT quantities well defined. Then all phenomena are well defined and normally explicable, that is, no room for anomaly and ill-definedness. In this sense, an anomaly is 'canonical' in the complete underlying theory of everything (CUTE). On the other hand, in the 'low energy' ranges corresponding to EFT's, the underlying degrees become formally decoupled. But their contributions might induce composite EFT operators, similar to the well-known Wilson OPE [8] and decoupling of heavy quark fields [9]. It is the appearance of these operators that become 'anomalies' in EFT, as they are not 'canonically' defined in the EFT formulations, but come from the decoupling limit of the underlying modes, or new physics.

A. EFT vs. CUTE: general formulation

To proceed in a formal way, we assume that it is still appropriate to use an abstract Hilbert space for CUTE where operators, coupling constants and state vectors are necessary theoretical components. We need not speculate about what they look like, we only need their existence to fill in the gap in the UV ends of EFT spectra. In other words, we regularize the EFT with the correct CUTE structures instead of an arbitrary regularization schemes. Then we could formally define the EFT generating functional (path integral) in CUTE, and the associated Green functions or vertex functions, by integrating out the CUTE modes $[\eta]$ with the CUTE constants left over, $\{\sigma\}$,

$$Z\{[J_{EFT}]\} \equiv \int D[\phi]D[\eta]exp\{i \int d^D x(\mathcal{L}_{CUTE}([\phi([\eta])]|[\eta],[\sigma]) + [J_{EFT} * \phi([\eta])]\})$$
$$= \int D[\phi_{\{\sigma\}}]exp\{i \int d^D x(\mathcal{L}_{EFT}([\phi_{\{\sigma\}}],[g]|\{\sigma\}) + [J_{EFT} * \phi_{\{\sigma\}}])\}, \qquad (1)$$

where $[J_{EFT}]$ denotes the external sources for EFT and $[g]$ are EFT couplings (which might be green functions in terms of CUTE parameters). In the first line the path integral are naturally factorized as the typical scales for EFT and CUTE should be widely separated.

The generating functional for vertex functions, upon field expansion, now takes the following form,

$$\Gamma_{EFT}([g],[\Phi_{\{\sigma\}}]|\{\sigma\}) = \sum \frac{1}{n!} \int \prod_{i=1}^n d^D x_i[\Phi_{\{\sigma\}}(x_i)]\Gamma^{(n)}([x_1, x_2, \cdots, x_n],[g]|\{\sigma\}), \qquad (2)$$

with $\Gamma^{(n)}([x_1, x_2, \cdots, x_n],[g]; \{\sigma\})$ being the n-point complete vertex functions that are well defined in CUTE. Removing the EFT loop contributions we get the EFT Lagrangian (tree vertices),

$$\mathcal{L}_{EFT}([g],[\Phi_{\{\sigma\}}]|\{\sigma\}) \equiv \Gamma_{EFT}([g],[\Phi_{\{\sigma\}}]|\{\sigma\})|_{\text{loops removed}}. \qquad (3)$$

As no EFT loops are involved such EFT object is well defined in the low energy (LE) limit $\mathbf{L}^{\{\sigma\}}$: $\frac{\Lambda_{EFT}}{\Lambda_{CUTE}} \Longrightarrow 0$,

$$\mathcal{L}_{EFT}([g], [\Phi]) \equiv \mathbf{L}^{\{\sigma\}} \mathcal{L}_{EFT}([g], [\Phi_{\{\sigma\}}]|\{\sigma\}), \tag{4}$$

that means the CUTE modes or new physics are completely decoupled from EFT at the Lagrangian or tree level (and in certain convergent loop diagrams). But this does not automatically lead to the complete decoupling in certain set of EFT loops due to ill-definedness, because the low energy limit operation and the EFT loop integration (summation over intermediate states) do not commute on these amplitudes,

$$[\mathbf{L}^{\{\sigma\}}, \int_{\text{loop or path}}] = [\mathbf{L}^{\{\sigma\}}, \sum_{\text{intermediate states}}] \neq 0. \tag{5}$$

The correct way is to carry out the loop integration first[2].

One might contend that the preceding simple arguments seems useless in practice as CUTE (or new physics) are unknown at all. We disagree with this point of view. In next subsection we demonstrate a further and, we think, nontrivial use of the regularity of CUTE or its existence in computing the EFT loop amplitudes.

B. differential equations following from the existence of underlying theory

Let us illustrate the method on a one loop Feynman diagram γ that is divergent in EFT parametrization. In CUTE definition it should take the following form,

$$\Gamma_\gamma([p], [g]|\{\sigma\}) = \int d^4k f_\gamma(k, [p], [g]|\{\sigma\}). \tag{6}$$

To determine its low energy limit we note the following fact [11,12]: Differentiating a divergent amplitude with respect to external parameters (momenta and masses) reduces and even removes the divergence. In CUTE language, an EFT loop's dependence upon $\{\sigma\}$ is reduced or completely removed (CUTE effects completely decoupled) if appropriate times of differentiation are done. For the one loop example $\Gamma_\gamma([p], [g]|\{\sigma\})$, this is,

$$\partial_{[p]}^{\omega_\gamma} \{\mathbf{L}^{\{\sigma\}} \Gamma_\gamma([p], [g]|\{\sigma\})\} = \mathbf{L}^{\{\sigma\}} \{\int d^4k \partial_{[p]}^{\omega_\gamma} f_\gamma(k, [p], [g]|\{\sigma\})\}$$

$$= \int d^4k \partial_{[p]}^{\omega_\gamma} \{\mathbf{L}^{\{\sigma\}} f_\gamma(k, [p], [g]|\{\sigma\})\} = \int d^4k \partial_{[p]}^{\omega_\gamma} f_\gamma(k, [p], [g]) \equiv \Gamma_\gamma^{(\omega_\gamma)}([p], [g]). \tag{7}$$

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[2]One might argue that the low energy limit might again lead to UV divergences. To respond, we note that ill-definedness is with our knowledge, but not with the nature. Physical theories keep becoming better defined as more underlying physics are uncovered, recall the evolution from Fermi's β decay theory to the electroweak theory. In fact, there does exist a mathematical framework that could yield finite results without subtraction [10]. Here we wish to explore the physical regularity masked by the ill-definedness that might be more appealing to physicists.

The above derivation is based on and guaranteed by the regularity of the CUTE definition, a nontrivial use of the existence of CUTE as stated above. $\omega_\gamma - 1$ is set equal to the superficial degree of γ according to Weinberg theorem [13]. Then the loop integration can be safely performed within EFT.

Technically, this method is to obtain the CUTE version of EFT amplitudes from the following kind of inhomogeneous differential equations,

$$\partial_{[p]}^{\omega_\gamma} Y_\gamma([p],[g];\{C\}) = \Gamma_\gamma^{(\omega_\gamma)}([p],[g]), \tag{8}$$

with $\{C\}$ representing the ambiguities inherent in differential equation solutions. So we can only determine the quantities we want up to a polynomial,

$$\mathbf{L}^{\{\sigma\}}\Gamma_\gamma([p],[g]|\{\sigma\}) = Y_\gamma([p],[g];\{C\})\mathrm{mod}\mathcal{P}^{(\omega_\gamma - 1)}, \tag{9}$$

with $\mathcal{P}^{(\omega_\gamma - 1)}$ being the polynomial. Of course the constants $\{C\}$ should be uniquely defined in CUTE. But the validity of differential equations derived above implies that: the decoupling effects from CUTE or new physics must lie in the space spanned by $\{C\}$, *no matter what they are*. In this sense, the differential equation approach is the most general way for parametrizing the possible form of CUTE or new physics in the decoupling limit.

In principle, fixing the constants $\{C\}$ according to physical (boundary) conditions or experimental data, just like what we do in classical electrodynamics and quantum mechanics[3], can spare the procedure of intermediate subtractions and lead to a simple and natural strategy for renormalization [15]. The application to multi-loop cases is straightforward, see Ref. [15] for detailed description. A nontrivial two loop example can be found in Ref. [16]. The annoying overlapping divergences are automatically dissolved in this differential equation approach [12]. From now on we temporarily call the strategy the differential equation approach.

Graphically, the differentiation is to insert scalar ($\partial_{[\mathrm{mass}]}$) or vector ($\partial_{[\mathrm{momentum}]}$) EFT vertex (with zero momentum transfer $q = 0$) into the diagrams, which lowers their degree of superficial divergence, see Fig.1.
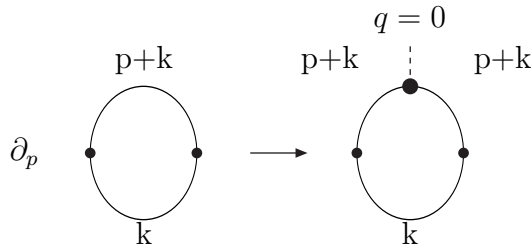

Fig.1. Differentiation w.r.t. external momentum

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[3]It is easy to see that the constants thus determined must be, in the conventional renormalization terminology, scheme and scale invariant [14].

C. Two simple examples

First let us consider the following simple one fermion loop amplitude,

$$\Gamma_{1\nu} = 2m \int \frac{d^4l}{(2\pi)^4} \mathrm{tr}(\frac{i}{\gamma_\alpha l^\alpha - m} \gamma_\nu \gamma_5 \frac{i}{\gamma_\alpha (l-p)^\alpha - m} \gamma_5)$$

$$= -8m^2 p_\nu \int \frac{d^4l}{(2\pi)^4} \frac{1}{[l^2 - m^2][(l-p)^2 - m^2]} \equiv -8m^2 p_\nu I. \qquad (10)$$

This loop has a linear superficial, twice differentiation with respect to p or m are enough.

Technically, the differentiation can be performed either at the graph level or at the reduced integral level (i.e., on I), provided Lorentz invariance are required. Usually (see, Sec. III), a fermion loop amplitude would give rise to a sum of integrals with different divergences, some are even convergent, thus it will be time consuming to perform the differentiation at the graph level. In this example, the true divergence in I is only logarithmic, and once differentiation is OK. Therefore we choose to differentiate at the level of the reduced integral (I),

$$\partial_{p^\mu} I = 2 \int \frac{d^4l}{(2\pi)^4} \frac{(l-p)_\mu}{[l^2 - m^2][(l-p)^2 - m^2]^2} \equiv 2I_{1;\mu} - 2p_\mu I_2. \qquad (11)$$

Now the two integrals $I_{1;\mu}$ and I_2 are well defined within EFT and the loop integration can be done,

$$I_{1;\mu} = -\frac{ip_\mu}{(4\pi)^2} \int_0^1 dx \frac{(1-x)^2}{m^2 + p^2(x^2 - x)}, \quad I_2 = -\frac{i}{(4\pi)^2} \int_0^1 dx \frac{1-x}{m^2 + p^2(x^2 - x)}. \qquad (12)$$

Then we have,

$$\partial_{p^\mu} I = -2 \frac{ip_\mu}{(4\pi)^2} \int_0^1 dx \frac{(x^2 - x)}{m^2 + p^2(x^2 - x)} = \partial_{p^\mu} \{-\frac{i}{(4\pi)^2} \int_0^1 dx \ln(m^2 + p^2(x^2 - x))\}, \qquad (13)$$

from which the solution can be readily found as

$$I = -\frac{i}{(4\pi)^2} [\int_0^1 dx \ln \frac{m^2 + p^2(x^2 - x)}{\mu^2} + C], \qquad (14)$$

$$\Longrightarrow \Gamma_{1\nu} = \frac{im^2 p_\nu}{2\pi^2} [\int_0^1 dx \ln \frac{m^2 + p^2(x^2 - x)}{\mu^2} + C], \qquad (15)$$

with two unknown constants: one dimensional,μ, the other dimensionless C. It is easy to see that they should be independent of the tree parameter m: $\partial_m \mu = \partial_m C = 0$.

As mentioned above, differentiation at the graph level should lead to the same expression, provided Lorentz invariance is imposed on the amplitude. Therefore, Lorentz non-invariant constants like C^ν are discarded. Of course, under certain circumstances, such kind of constants might be allowed and useful.

As the second example we calculate the following one loop amplitude,

6

$$\Gamma_2 = 4m^2 \int \frac{d^4l}{(2\pi)^4} \text{tr}(\frac{i}{\gamma_\alpha l^\alpha - m} m \frac{i}{\gamma_\alpha l^\alpha - m} \gamma_5 \frac{i}{\gamma_\alpha (l-p)^\alpha - m} \gamma_5) = 16im^4(I_3 - p^2 I_4),$$

$$I_3 = \int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2 - m^2)^2}, \quad I_4 = \int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2 - m^2)^2((l-p)^2 - m^2)}. \tag{16}$$

Of the two integrals only I_3 has a logarithmic divergence, I_4 is convergent. So we only need to differentiate I_3 once, with respect to mass. However, noting that $I_3 = I|_{p=0}$, we could make use of the preceding calculation to obtained that $I_3 = -\frac{i}{(4\pi)^2}[\ln \frac{m^2}{\mu^2} + C]$. Then with $I_4 = -\frac{i}{2(4\pi)^2} \int_0^1 dx \frac{1}{m^2 + p^2(x^2 - x)}$ we have,

$$\Gamma_2 = \frac{m^2}{2\pi^2}\{2m^2(\ln \frac{m^2}{\mu^2} + C) - p^2 \int_0^1 dx \frac{1}{m^2 + p^2(x^2 - x)}\}. \tag{17}$$

Again, we stress that this is the most general parametrization of the two amplitudes that are Lorentz invariant. From the above examples we could also see that there is no subtlety like the definition of metric tensor and γ_5 that is associated with dimensional schemes [17].

III. CALCULATION AND PARAMETRIZATION OF TWO POINT FUNCTIONS OF AXIAL CURRENT AND ITS DIVERGENCE

For simplicity we work in $U(1)$ gauge theory with just one massive fermion, i.e., QED. The objects to be computed and investigated are respectively [5]

$$\Pi^5_{\mu\nu}(p,-p) \equiv i\mathcal{FT}\{\langle 0|T(j_{5\mu}j_{5\nu})|0\rangle\}, \quad \Delta^5_{\mu\nu}(0,p,-p) \equiv \mathcal{FT}\{\langle 0|T(\theta j_{5\mu}j_{5\nu})|0\rangle\}; \tag{18}$$

$$\Pi^5_{\ \nu}(p,-p) \equiv i\mathcal{FT}\{\langle 0|T(j_5 j_{5\nu})|0\rangle\}, \quad \Delta^5_{\ \nu}(0,p,-p) \equiv \mathcal{FT}\{\langle 0|T(\theta j_5 j_{5\nu})|0\rangle\}; \tag{19}$$

$$\Pi^5(p,-p) \equiv i\mathcal{FT}\{\langle 0|T(j_5 j_5)|0\rangle\}, \quad \Delta^5(0,p,-p) \equiv \mathcal{FT}\{\langle 0|T(\theta j_5 j_5)|0\rangle\}; \tag{20}$$

$$\langle\sigma\rangle \equiv 4m\langle\bar\psi\psi\rangle, \quad \Pi^{\theta\sigma}(0,0) \equiv -i\mathcal{FT}\{\langle 0|\theta\sigma|0\rangle\}, \tag{21}$$

$$j^5_\mu \equiv \bar\psi\gamma_\mu\gamma_5\psi, \quad j^5 \equiv 2im\bar\psi\gamma_5\psi, \quad \theta \equiv m\bar\psi\psi, \quad \sigma \equiv 4m\bar\psi\psi. \tag{22}$$

where $\mathcal{FT}\{\cdots\}$ denotes the Fourier transform and m refers to the fermion mass. At one loop level they read (see Fig.1)

$$\Pi^5_{\mu\nu}(p,-p) = -i \int \frac{d^4l}{(2\pi)^4} \text{tr}(\frac{i}{A}\gamma_\nu\gamma_5 \frac{i}{B}\gamma_\mu\gamma_5), \tag{23}$$

$$\Delta^5_{\mu\nu}(0,p,-p) = -\int \frac{d^4l}{(2\pi)^4} \text{tr}(\frac{i}{A}m\frac{i}{A}\gamma_\nu\gamma_5 \frac{i}{B}\gamma_\mu\gamma_5) + \text{cross term}; \tag{24}$$

$$\Pi^5_{\ \nu}(p,-p) = 2m \int \frac{d^4l}{(2\pi)^4} \text{tr}(\frac{i}{A}\gamma_\nu\gamma_5 \frac{i}{B}\gamma_5), \tag{25}$$

$$\Delta^5_{\ \nu}(0,p,-p) = -2im \int \frac{d^4l}{(2\pi)^4} \text{tr}(\frac{i}{A}m\frac{i}{A}\gamma_\nu\gamma_5 \frac{i}{B}\gamma_5) + \text{cross term}; \tag{26}$$

$$\Pi^5(p,-p) = 4m^2 i \int \frac{d^4l}{(2\pi)^4} \text{tr}(\frac{i}{A}\gamma_5 \frac{i}{B}\gamma_5), \tag{27}$$

$$\Delta^5(0,p,-p) = 4m^2 \int \frac{d^4l}{(2\pi)^4} \text{tr}(\frac{i}{A}m\frac{i}{A}\gamma_5 \frac{i}{B}\gamma_5) + \text{cross term}; \tag{28}$$

7

$$\langle \sigma \rangle = -4m \int \frac{d^4l}{(2\pi)^4} \text{tr}(\frac{i}{A}), \quad \Pi^{\theta\sigma}(0,0) = 4im^2 \int \frac{d^4l}{(2\pi)^4} \text{tr}(\frac{i}{A}\frac{i}{A}), \tag{29}$$

$$A \equiv \gamma_\alpha l^\alpha - m, \quad B \equiv \gamma_\alpha (l-p)^\alpha - m. \tag{30}$$

It is easy to carry out the Dirac traces to arrive at the following form

$$\Pi^5_{\mu\nu}(p,-p) = 2ig_{\mu\nu}((p^2 - 4m^2)I_{ab} - I_a - I_b) + 8iI_{ab;\mu\nu} - 4ip_\mu I_{ab;\nu} - 4ip_\nu I_{ab;\mu}, \tag{31}$$

$$\Delta^5_{\mu\nu}(0,p,-p) = 8im^2 g_{\mu\nu}(p^2 I_{a^2b} - I_{a^2} - 2I_{ab} - 4m^2 I_{a^2b}) + 32im^2 I_{a^2b;\mu\nu}$$
$$\quad -16im^2(p_\mu I_{a^2b;\nu} + p_\nu I_{a^2b;\mu}); \tag{32}$$

$$\Pi^5_\nu(p,-p) = -8m^2 I_{ab}p_\nu, \tag{33}$$

$$\Delta^5_\nu(0,p,-p) = 16m^2 I_{ab;\nu} - 16m^2(I_{ab} + 2m^2 I_{a^2b})p_\nu; \tag{34}$$

$$\Pi^5(p,-p) = 8im^2(I_a + I_b - p^2 I_{ab}), \tag{35}$$

$$\Delta^5(0,p,-p) = 32im^4(I_{a^2} - p^2 I_{a^2b}); \tag{36}$$

$$\langle \sigma \rangle = -16im^2 I_a, \quad \Pi^{\theta\sigma}(0,0) = \langle \sigma \rangle - 32im^4 I_{a^2}, \tag{37}$$

$$I_{a^n b^k} \equiv \int \frac{d^4l}{(2\pi)^4} \frac{1}{a^n b^k}, \quad I_{a^n b^k;\mu} \equiv \int \frac{d^4l}{(2\pi)^4} \frac{l_\mu}{a^n b^k}, I_{a^n b^k;\mu\nu} \equiv \int \frac{d^4l}{(2\pi)^4} \frac{l_\mu l_\nu}{a^n b^k}, \tag{38}$$

$$a \equiv l^2 - m^2, \quad b \equiv (l-p)^2 - m^2. \tag{39}$$

Most of the integrals are UV ill-defined and hence can not be directly performed.

To calculate these integrals we will employ two different methods as mentioned in the introduction, one is the most frequently used dimensional regularization, another is the differential equation approach introduced in sec. II. It is easy to perform the calculations in the two approaches and the parametrization of the integrals will be given in the appendix A. With these integrals the two- and three- point functions read

$$\Pi^5_{\mu\nu}(p,-p) = \frac{2g_{\mu\nu}}{(4\pi)^2}\{p^2[2C_0 - C_4 + \Delta_0 + 2\int_0^1 da(x^2 - x)(\ln\frac{D}{\mu^2} + C_4 - 1)]$$

$$-2m^2[\Delta_0 + \ln\frac{m^2}{\mu^2} + 2C_0] + 2C' + 2C'_5\}$$

$$+\frac{p_\mu p_\nu}{4\pi^2}\{2\int_0^1 da(1-x)^2[\ln\frac{D}{\mu^2} + C_4] - C_0 - \Delta_0\}, \tag{40}$$

$$\Delta^5_{\mu\nu}(0,p,-p) = \frac{m^2 p_\mu p_\nu}{\pi^2 p^2}(1 - \frac{m^2}{\Delta}) - \frac{g_{\mu\nu}m^2}{2\pi^2}\{\ln\frac{m^2}{\mu^2} + 2C_0 + \Delta_0 + \frac{2m^2 - p^2/2}{\Delta}\}; \tag{41}$$

$$\Pi^5_\nu(p,-p) = \frac{im^2 p_\nu}{2\pi^2}(\Delta_0 + C_0), \tag{42}$$

$$\Delta^5_\nu(0,p,-p) = \frac{im^2 p_\nu}{2\pi^2}(\Delta_0 + C_0 + \frac{2m^2}{\Delta}); \tag{43}$$

$$\Pi^5(p,-p) = \frac{m^2}{2\pi^2}\{2m^2(\ln\frac{m^2}{\mu^2} + C_0 - 1) + 2C' - p^2(\Delta_0 + C_0)\}, \tag{44}$$

$$\Delta^5(0,p,-p) = \frac{m^2}{\pi^2}\{2m^2(\ln\frac{m^2}{\mu^2} + C_0) - \frac{p^2}{\Delta}\}; \tag{45}$$

$$\langle \sigma \rangle = \frac{m^2}{\pi^2}\{m^2(1 - \ln\frac{m^2}{\mu^2} - C_0) - C'\}, \tag{46}$$

8

$$\Pi^{\theta\sigma}(0,0) = \frac{m^2}{\pi^2}\{3m^2(1/3 - \ln\frac{m^2}{\mu^2} - C_0) - C'\}. \tag{47}$$

For comparison we also list the results in dimensional regularization here,

$$\Pi^{5,\epsilon}_{\mu\nu}(p,-p) = \frac{2g_{\mu\nu}}{(4\pi)^2}\{p^2[\Delta_0 - \Gamma(\epsilon) + 2\int_0^1 da(x^2 - x)(\ln\frac{D}{4\pi\mu^2} - \Gamma(\epsilon) - 1)]$$

$$-2m^2[\Delta_0 + \ln\frac{m^2}{4\pi\mu^2} - 2\Gamma(\epsilon)]\}$$

$$+\frac{p_\mu p_\nu}{4\pi^2}\{2\int_0^1 da(1-x)^2[\ln\frac{D}{4\pi\mu^2} - \Gamma(\epsilon)] + \Gamma(\epsilon) - \Delta_0\}, \tag{48}$$

$$\Delta^{5,\epsilon}_{\mu\nu}(0,p,-p) = \frac{g_{\mu\nu}m^2}{2\pi^2}\{2\Gamma(\epsilon) - \ln\frac{m^2}{4\pi\mu^2} - \Delta_0 + \frac{p^2/2 - 2m^2}{\Delta}\} + \frac{m^2 p_\mu p_\nu}{\pi^2 p^2}(1 - \frac{m^2}{\Delta}); \tag{49}$$

$$\Pi^{5,\epsilon}_{\;\nu}(p,-p) = \frac{im^2 p_\nu}{2\pi^2}(\Delta_0 - \Gamma(\epsilon)), \tag{50}$$

$$\Delta^{5,\epsilon}_{\;\nu}(0,p,-p) = \frac{im^2 p_\nu}{2\pi^2}(\Delta_0 - \Gamma(\epsilon) + \frac{2m^2}{\Delta}); \tag{51}$$

$$\Pi^{5,\epsilon}(p,-p) = \frac{m^2}{2\pi^2}\{2m^2(\ln\frac{m^2}{4\pi\mu^2} - \Gamma(\epsilon) - 1) - p^2(\Delta_0 - \Gamma(\epsilon))\}, \tag{52}$$

$$\Delta^{5,\epsilon}(0,p,-p) = \frac{m^2}{\pi^2}\{2m^2(\ln\frac{m^2}{4\pi\mu^2} - \Gamma(\epsilon)) - \frac{p^2}{\Delta}\}; \tag{53}$$

$$\langle\sigma\rangle^\epsilon = \frac{m^4}{\pi^2}(\Gamma(\epsilon) + 1 - \ln\frac{m^2}{4\pi\mu^2}), \tag{54}$$

$$\Pi^{\theta\sigma,\epsilon}(0,0) = \frac{3m^4}{\pi^2}(\Gamma(\epsilon) + 1/3 - \ln\frac{m^2}{\mu^2}). \tag{55}$$

It is easy to see that setting $C_0 = C_4 = -\Gamma(\epsilon), C' = C_5' = 0$ in the differential equation solutions we will arrive at dimensional regularization for these integrals. Again we see that the differential equation approach is a general parametrization. Now we turn to the next section to study the trace and chiral identities.

IV. TRACE AND CHIRAL WARD IDENTITIES AND THEIR DETERMINANTS

A. Canonical identities and general parametrization of anomalies

First Let us write down the canonical identities for trace relation and chiral symmetry that should satisfied by the above vertex functions [5]:

$$\Delta^5_{\mu\nu}(0,p,-p) = (2 - p\partial_p)\Pi^5_{\mu\nu}(p,-p), \tag{56}$$

$$\Delta^5_{\;\nu}(0,p,-p) = (2 - p\partial_p)\Pi^5_{\;\nu}(p,-p), \tag{57}$$

$$\Delta^5(0,p,-p) = (2 - p\partial_p)\Pi^5(p,-p); \tag{58}$$

$$-ip^\mu\Delta^5_{\mu\nu}(0,p,-p) = \Delta^5_{\;\nu}(0,p,-p) + \Pi^5_{\;\nu}(p,-p), \tag{59}$$

$$ip^\nu\Delta^5_{\;\nu}(0,p,-p) = \Delta^5(0,p,-p) + \Pi^5(p,-p) + \Pi^{\theta\sigma}(0,0), \tag{60}$$

$$-ip^\mu\Pi^5_{\mu\nu}(p,-p) = \Pi^5_{\;\nu}(p,-p), \tag{61}$$

$$ip^\nu\Pi^5_{\;\nu}(p,-p) = \Pi^5(p,-p) + \langle\sigma\rangle. \tag{62}$$

The first three are trace identities and the other four are chiral Ward identities.

Now let us check them with our results given above. Again we should stress that in the differential equation approach, we have parametrize the regularization dependence or CUTE decoupling effects in a most general way that is consistent with Lorentz invariance. If one choose not to obey Lorentz invariance, then more constants that not Lorentz invariant like $C_\mu, C_{\mu\nu}$ should appear. This might be possible choices under certain circumstances. Here we assume there is no violation of Lorentz invariance in any form of CUTE and its low energy limits. After simple algebra, we find that in the general parametrization of the CUTE decoupling (or low energy limit) effects, Eq.s(56, 57, 58, 61) are modified as follows,

$$\Delta^5_{\mu\nu}(0,p,-p) = (2 - p\partial_p)\Pi^5_{\mu\nu}(p,-p) + \frac{1}{6\pi^2}(g_{\mu\nu}p^2 - p_\mu p_\nu) - \frac{g_{\mu\nu}}{\pi^2}(m^2 + (C' + C'_5)/2), \quad (63)$$

$$\Delta^5_{\ \nu}(0,p,-p) = (2 - p\partial_p)\Pi^5_{\ \nu}(p,-p) + \frac{im^2 p_\nu}{\pi^2}, \quad (64)$$

$$\Delta^5(0,p,-p) = (2 - p\partial_p)\Pi^5(p,-p) - \frac{m^2 p^2}{\pi^2} + \frac{2m^2}{\pi^2}(m^2 - C'); \quad (65)$$

$$-ip^\mu \Pi^5_{\mu\nu}(p,-p) = \Pi^5_{\ \nu}(p,-p) - \frac{i(C' + C'_5)p_\nu}{4\pi^2}. \quad (66)$$

Here we see that anomalies appear in all the three trace identities and one chiral Ward identities. But chiral anomaly in Eq.(66) is regularization scheme or underlying effects dependent, as long as $C' + C'_5 = 0$ the chiral symmetry is restored in the two point functions, that is, not all regularization schemes violate Eq.(61), or equivalently, such anomaly is sensitively dependent on the underlying theory details, and the existence of such anomaly is consistent with the trace identities with anomalies, see below. This is a modification of Wilson's argument for two point functions [5]. However, the anomaly in the trace identities can not be removed in any regularization, or equivalently, the CUTE decoupling effects do violate the EFT trace identities.

B. Chiral anomaly and consistency

Now let us check the consistency among the chiral and trace identities with anomalies. First let us consider the case where there is no chiral anomaly in Eq.(61), i.e., $C' + C'_5 = 0$. Then applying the relations Eq.s(59, 60, 61, 62) to the trace identities (63, 64), we get

$$\Delta^5_{\mu\nu}(0,p,-p) = (2 - p\partial_p)\Pi^5_{\mu\nu}(p,-p) + \frac{1}{6\pi^2}(g_{\mu\nu}p^2 - p_\mu p_\nu) - \frac{g_{\mu\nu}m^2}{\pi^2}, \quad (67)$$

$$\Delta^5_{\ \nu}(0,p,-p) = (2 - p\partial_p)\Pi^5_{\ \nu}(p,-p) + \frac{im^2 p_\nu}{\pi^2}, \quad (68)$$

$$\Delta^5(0,p,-p) = (2 - p\partial_p)\Pi^5(p,-p) - \frac{m^2 p^2}{\pi^2} + 3\langle\sigma\rangle - \Pi^{\theta\sigma}(0,0). \quad (69)$$

The second trace relation agrees exactly with Ref. [5], there are some disagreements in the other two: (1) in Ref. [5], the numerical coefficient of the anomaly term $(g_{\mu\nu}p^2 - p_\mu p_\nu)$ is $\frac{1}{8\pi^2}$ while in our first equation it is $\frac{1}{6\pi^2}$; (2) in Ref. [5] the last two terms in Eq.(69) were missing, we will discuss about this point later. Note that in Eq.(69), $\frac{2m^2}{\pi^2}(m^2 - C') = 3\langle\sigma\rangle - \Pi^{\theta\sigma}(0,0)$

is used. We should remind that this term can not be removed by setting $C' = m^2$ as the constant C' is independent of m: $\partial_m C' = 0$. Thus putting $C' + C'_5 = 0$ we can conclude that in any CUTE whose decoupling limit preserve the chiral Ward identities for two point functions, the trace anomalies at one loop level for the two point functions are consistently given in Eq.s(67, 68, 69), no matter what numerical values C_0, C_4 may take. Or one can claim that as long as the new physics in the low energy limit does not violate chiral symmetry in the two point functions, then the trace relation at one loop level for these two point functions are consistently dictated by these three equations.

Now let us consider the case where $C' + C'_5 \neq 0$. Imposing the chiral relations Eq.s(59, 66) on Eq.(63) we have,

$$\Delta^5{}_\nu(0, p, -p) = (2 - p\partial_p)\Pi^5{}_\nu(p, -p) + \frac{im^2 p_\nu}{\pi^2}, \tag{70}$$

which is exactly Eq.(64). Then imposing Eq.s(60, 62) on Eq.(64) we find that

$$\Delta^5(0, p, -p) = (2 - p\partial_p)\Pi^5(p, -p) - \frac{m^2 p^2}{\pi^2} + 3\langle\sigma\rangle - \Pi^{\theta\sigma}(0, 0), \tag{71}$$

again in agreement with Eq.(65) or Eq. (69). This completes the consistency check on the identities (Eq.s (63), (64), (65), (59), (60), (66) and (62)) in the most general parametrization of the decoupling effects from CUTE or new physics.

In other words, in the regularization schemes or CUTE decoupling limit where $C' + C'_5 \neq 0$, the chiral Ward identities (with anomalies) are also consistent with the trace identities (with anomalies) for the two point functions, at least at one loop level. It is not necessary to impose chiral symmetry in the Ward identities (59), (60), (61) and (62) so that $C' + C'_5 = 0$. That is the chiral schemes (where $C' + C'_5 = 0$) are not mathematically superior to the ones violating chiral symmetry (where $C' + C'_5 \neq 0$). Of course, the final determination of what value $C' + C'_5$ should take must be searched in physical 'data'.

Obviously dimensional regularization is a scheme preserving chiral symmetry, where $C' + C'_5 = 0$. In fact $C' = C'_5 = 0$ in dimensional regularization. It is just a special way of parametrizing the divergence which might become unfavorable and should be altered in certain situations [18], for the correspondence between the DR poles and the cutoff powers, see Ref. [19].

C. Remarks on Eq.(69)

Now let us return to Eq.(69) or (65). From the trace identity perspective, both $-\frac{m^2 p^2}{\pi^2}$ and $3\langle\sigma\rangle - \Pi^{\theta\sigma}(0, 0)$ are anomalies (we have explained in the preceding subsection that the latter could not be removed by setting $C' = m^2$). But the latter is required by and explicable within chiral Ward identities and its existence is independent of CUTE or new physics, thus we arrive at an interesting phenomenon: *the canonical terms in chiral identity become anomalies in trace identity.* Thus it seems that EFT chiral symmetry is quite different with EFT scaling laws: the former could be preserved by new physics or CUTE (i.e., new physics or CUTE heavy modes could be completely decoupled from EFT chiral identities ($C' + C'_5 = 0$) provided the underlying contents are free of chiral anomaly everywhere [6]),

but the latter is inevitably modified by CUTE or new physics, that is, CUTE or new physics can never be completely decoupled from the EFT scaling laws even when chiral symmetry is perfectly preserved. This might lead to the speculation that CUTE or new physics and EFT's might not differ too much in the chiral symmetry perspective, but they should be essentially different in the scale symmetry.

As our investigation is done in a non-supersymmetric EFT, it would be interesting to see how this disparity between chiral and scale symmetry evolves as the theory turns super-symmetric, where the dilatation current and axial vector current consist a supersymmetric multiplet and should have the identical anomaly status [20].

This disparity might also be understood somehow from the non-renormalization of chiral anomaly which is known as the Adler-Bardeen theorem [21]. In CUTE language, this means that the CUTE modes only slightly modify the EFT chiral symmetry at the one loop level, they are completely decoupled from chiral symmetry in all the rest EFT amplitudes, or they are even completely decoupled in appropriate EFT contents [6]. But the CUTE modes significantly modify the EFT trace identities or scaling laws in all orders of the EFT loop amplitudes (for certain vertices) that are ill-defined, no matter to what degree chiral symmetry is affected. Thus the non-renormalization theorems of various objects and relations might lead to similar phenomenon, more examples might be found in supersymmetric contexts [22].

V. INTERPRETATION OF THE RESULTS

To interpret the trace anomalies in CUTE language, we first note that the full scaling law (trace identities) in CUTE version of the EFT vertex should read,

$$\Gamma^{(n)}([\lambda p], [\lambda^{d_g} g]; \{\lambda^{d_\sigma} \sigma\}) = \lambda^{d_{\Gamma^{(n)}}} \Gamma^{(n)}([p], [g]; \{\sigma\}), \tag{72}$$

where $d_{...}$ refers to the canonical scale dimension of a parameter or canonical mass dimension of a constant (effective and/or underlying), that is the vertex function **must** be a homogeneous function of all its dimensional arguments. In differential form, it is

$$\{\sum_{i=1}^{n} p_i \cdot \partial_{p_i} + \sum d_\sigma \sigma \partial_\sigma + \sum d_g g \partial_g - d_{\Gamma^{(n)}}\} \Gamma^{(n)}([p], [g]; \{\sigma\}) = 0, \tag{73}$$

where $d_{...}$ refers to the canonical scaling dimension.

Specifically, for the one-loop two point functions considered above Eq.(73) implies that

$$\{p\partial_p + \sum d_\sigma \sigma \partial_\sigma + m\partial_m - 2\} \Pi^{\cdots}(p, -p, m; \{\sigma\}) = 0$$
$$\implies \Delta^{\cdots}(p, 0, -p, m; \{\sigma\}) = (2 - p\partial_p - \sum d_\sigma \sigma \partial_\sigma) \Pi^{\cdots}(p, -p, m; \{\sigma\}), \tag{74}$$

where $\Delta^{\cdots}(p, 0, -p, m; \{\sigma\}) \equiv m\partial_m \Pi^{\cdots}(p, -p, m; \{\sigma\})$. Here, we have explicitly put mass m into the arguments of the two- and three- point functions. Obviously, Eq.s (73, 74) describe the 'canonical' scaling law (or trace identities) in CUTE language, no anomalies. Then, in the decoupling limit, if $\sum d_\sigma \sigma \partial_\sigma$'s contributions vanish, we say a complete decoupling of CUTE modes is realized and there is no anomaly in the EFT identities. However, as is shown above, at least for the two point functions under consideration, this term leads to a

non-vanishing polynomial in terms of the EFT parameters in the decoupling limit, or the local operators in terms of EFT fields, which, unexpected from the EFT deduction, become anomalies for EFT language:

$$\mathbf{L}^{\{\sigma\}}\{\sum d_\sigma \sigma \partial_\sigma \Pi^{\cdots}(p, -p, m; \{\sigma\})\} = \sum d_{\bar{c}} \bar{c} \partial_{\bar{c}} \Pi^{\cdots}(p, -p, m; \{\bar{c}\}) = \text{anomalies}, \qquad (75)$$

$$\Pi^{\cdots}(p, -p, m; \{\bar{c}\}) = \mathbf{L}^{\{\sigma\}} \Pi^{\cdots}(p, -p, m; \{\sigma\}), \qquad (76)$$

where $\{\bar{c}\}$ refers to the constants defined by the decoupling limit, a CUTE definition of the constants $\{C\}$ that are obtained from differential equations in EFT.

Noting that the non-decoupling of CUTE modes just implies the UV divergence or ill-definedness in EFT, we technically reproduced the conventional interpretation that trace anomalies or violation of scaling laws are due to UV divergence. As UV divergence is unphysical, here we propose a physical rationale for the anomaly phenomenon by providing a general parametrization of the CUTE or new physics decoupling effects, which accommodates all possible regularization schemes, as long as they are mathematically consistent. Thus, the dimensional transmutation is nothing but the transformation of the 'canonical' CUTE structures or new physics into the EFT anomalies in the decoupling limit.

Considering the fact that all the decoupling effects in a loop amplitude always appear in the local (polynomial in momentum space) part of each loop amplitude of a vertex, we can easily arrive at the following operatoral equality:

$$\mathbf{L}^{\{\sigma\}}\{\sum_{\{\sigma\}} d_\sigma \sigma \partial_\sigma\} = \sum_{\{\bar{c}\}} d_{\bar{c}} \bar{c} \partial_{\bar{c}} = \sum_{\{\mathcal{O}_i\}} \delta_{\mathcal{O}_i} \hat{I}_{\mathcal{O}_i} = \text{trace or scale anomalies} \qquad (77)$$

as $\sum_{\{\bar{c}\}} d_{\bar{c}} \bar{c} \partial_{\bar{c}}$ always induce the insertion of the local operators ($\{\mathcal{O}_i\}$) corresponding to the vertex functions in each loop component of any EFT Feynman diagram. Here it is easy to see that $\delta_{\mathcal{O}_i}$ should be the 'anomalous' dimension of the composite operator $\mathcal{O}_i$. Noting that parametrizing the constants $\{\bar{c}\}$ in terms of an independent scale μ and a number of dimensionless constants $\{\bar{c}_0\}$, Eq.(77) is in fact an operator form for RGE in CUTE language.

$$\mathbf{L}^{\{\sigma\}}\{\sum_{\{\sigma\}} d_\sigma \sigma \partial_\sigma\} = \mu \partial_\mu = \sum_{\{\mathcal{O}_i\}} \delta_{\mathcal{O}_i} \hat{I}_{\mathcal{O}_i} \Longrightarrow \mu \partial_\mu - \sum_{\{\mathcal{O}_i\}} \delta_{\mathcal{O}_i} \hat{I}_{\mathcal{O}_i} = 0. \qquad (78)$$

Note that here all the tree parameters are 'bare' and 'physical' in the sense that they should be defined by CUTE or physical boundary conditions or data, unlike in the conventional renormalization programs. Moreover, the composite operators $\{\mathcal{O}_i\}$ might or might not come from EFT Lagrangian, thus this equation is generally valid in any EFT, whether it is renormalizable or not in the conventional sense [23].

For the EFT generating functional defined in CUTE (Eq.(2)), we can easily write down the following scaling law or trace identity with the help of CUTE

$$\{\sum_{\{\Phi_{\{\sigma\}}\}} \int d^D x [(d_{\Phi_{\{\sigma\}}} - x \cdot \partial_x)\Phi_{\{\sigma\}}(x)] \frac{\delta}{\delta \Phi_{\{\sigma\}}(x)} + \sum d_g g \partial_g + \sum d_\sigma \sigma \partial_\sigma - D\}$$

$$\times \Gamma_{EFT}([g], [\Phi_{\{\sigma\}}]|\{\sigma\}) = 0, \qquad (79)$$

$$\Longrightarrow \{\sum_{\{\Phi\}} \int d^D x [(d_\Phi - x \cdot \partial_x)\Phi(x)] \frac{\delta}{\delta \Phi(x)} + \sum_{[g]} d_g g \partial_g + \sum_{\{\bar{c}\}} d_{\bar{c}} \bar{c} \partial_{\bar{c}} - D\}$$

$$\times \Gamma^{1PI}([\Phi], [g]; \{\bar{c}\}) = 0. \qquad (80)$$

13

with D denoting the spacetime dimension. This equation holds for any consistent EFT's. Since $\sum_{[g]} d_g g \partial_g =$ canonical trace tensor $= g_{\mu\nu} \Theta^{\mu\nu}$ in any EFT, then in operator form, Eq.(80) reads

$$g_{\mu\nu} \hat{\Theta}^{\mu\nu} = \text{canonical trace} + \sum_{\mathcal{O}_i} \delta_{\mathcal{O}_i} \mathcal{O}_i, \qquad (81)$$

where $\sum_{\{\mathcal{O}_i\}} \delta_{\mathcal{O}_i} \mathcal{O}_i$ consist the trace anomaly coming from the decoupling limit of the 'canon-ical' CUTE 'trace' term: $\mathbf{L}^{\{\sigma\}} \sum_{\{\sigma\}} d_\sigma \sigma \partial_\sigma$. Thus the formal derivation of trace anomaly [24] in the CUTE language is very simple. The remaining work is to find the relevant loop diagrams for the n-point functions that are ill-defined in EFT.

Similarly, we could anticipate that, for chiral identities in CUTE version, we might have, for example, corresponding to Eq.(61),

$$ip^\mu \Pi^5_{\mu\nu}(p, -p|\{\sigma\}) = \Pi^5_{\ \nu}(p, -p|\{\sigma\}) + \text{terms normal in CUTE}, \qquad (82)$$

which becomes, in the decoupling limit, the following form,

$$ip^\mu \Pi^5_{\mu\nu}(p, -p) = \Pi^5_{\ \nu}(p, -p) + \mathbf{L}^{\{\sigma\}}\{\text{terms normal in CUTE}\}. \qquad (83)$$

The last term, if does not vanish in the decoupling limit, constitutes the anomaly to the chiral identity. This is just the CUTE extension of the fact that decoupling a heavy fermion leads to chiral anomaly in the triangle diagrams. In regularization language, the Pauli-Villars regulator could serve as a crude but intuitive substitute for the CUTE decoupling interpretation of chiral anomaly [3]. In gauge theories without chiral coupling and in $\lambda\phi^4$ [25], the trace anomaly could also be obtained through heavy matter decoupling [26].

Now we might speculate that all the EFT anomalies could be understood from the perspective of the underlying theory or new physics. Working with the differential equation approach, one could have a general parametrization of all the possible underlying theory's effects and discuss the issues without the limitations associated with a specific regularization to arrive at general conclusions.

VI. DISCUSSIONS AND SUMMARY

Now let us address some important issues.

Although the philosophy followed here is well known, we make further use of the philosophy in that we interpret it as the existence of a complete formulation of the theory that underlies all the EFT's and is well-defined in all respects, especially in the high energy region that is above the EFT domains. From this interpretation, one could derive a natural differential equation approach for computing the loop diagrams, which is in fact a general way for parametrizing all the possible underlying theory's decoupling effects (or all possible consistent regularization schemes).

In this sense, though each regularization is artificial and defected somehow, the necessity in employing one in calculating the divergent loops (or the necessity in introducing subtractions, say in BPHZ, which is often viewed as regularization independent), can be naturally understood in CUTE as a profound and physical phenomenon: the non-decoupling of the

CUTE details in the EFT loops, though they are completely decoupled from the EFT Lagrangian. The origin of UV divergence is identified in the CUTE language (or the EFT philosophy) as the non-commutativity of the operation of decoupling the CUTE modes and the EFT loop integration (or equivalently the summation over EFT intermediate states).

In such conception, we can easily discuss the anomalies (in the two point functions under consideration) in a general parametrization, and the anomalies can be naturally interpreted as the non-vanishing decoupling effects of the 'canonical' CUTE contributions. Moreover, we have shown in Sec. III that, no matter how CUTE or new physics affected the EFT chiral symmetry, the chiral Ward identities and trace identities are consistent with each other. On the other hand, our CUTE or new physics interpretation of the anomalies also implies the requirements for 'building' models for CUTE or 'identifying' new physics: they must yield the same EFT anomalies in the decoupling limit! In other words, the anomalies in EFT are just the simplified parametrization of the underlying structures, a generalization of 't Hooft's interpretation of chiral anomaly [6].

In fact one can exploit the underlying theory scenario and the decoupling behavior in the low energy regions beyond the interpretations of anomalies. One could manipulate a simple proof of the finiteness or non-renormalization of the Chern-Simons action by employing gauge theory with heavy fermions as the underlying theory, see Ref. [27].

In summary, we described a general parametrization of the ill-defined Feynman loop amplitudes that is based on the existence of a complete theory underlying the EFT's and investigated the trace identities and chiral identities for certain two-point functions. The anomalies are interpreted in the underlying theory or new physics' perspective. As by-products, we showed that: (1) trace identities are consistent with chiral identities no matter what kind of regularization schemes are used: preserving chiral symmetry or not; (2) some terms canonical in chiral identities are anomalous in trace identities. Related remarks and speculations were also presented.

ACKNOWLEDGEMENT

APPENDIX A

Dimensional regularization:

$$I_a^\epsilon = \frac{im^2}{(4\pi)^2}(\Gamma(\epsilon) + 1 - \ln\frac{m^2}{4\pi\mu^2}) = I_b^\epsilon, \tag{84}$$

$$I_{a^2}^\epsilon = \frac{i}{(4\pi)^2}(\Gamma(\epsilon) + 1 - \ln\frac{m^2}{4\pi\mu^2}), \tag{85}$$

$$I_{ab}^{\epsilon} = \frac{i}{(4\pi)^2}(\Gamma(\epsilon) - \Delta_0), \tag{86}$$

$$I_{ab;\mu}^{\epsilon} = \frac{ip_\mu}{2(4pi)^2}(\Gamma(\epsilon) - \Delta_0), \tag{87}$$

$$I_{ab;\mu\nu}^{\epsilon} = \frac{ig_{\mu\nu}}{2(4\pi)^2}(2p^2 \int_0^1 dx(x^2 - x)(\Gamma(\epsilon) + 1 - \ln\frac{D}{4\pi\mu^2}) + m^2(\Gamma(\epsilon) + 1 - \Delta_0))$$
$$+ \frac{ip_\mu p_\nu}{(4\pi)^2} \int_0^1 dx(1-x)^2(\Gamma(\epsilon) - \ln\frac{D}{4\pi\mu^2}), \tag{88}$$

$$I_{a^2b} = \frac{-i}{2(4\pi)^2\Delta}, \tag{89}$$

$$I_{a^2b;\mu} = \frac{ip_\mu}{(4\pi)^2p^2}(1 - \frac{m^2}{\Delta}), \tag{90}$$

$$I_{a^2b;\mu\nu}^{\epsilon} = \frac{ip_\mu p_\nu}{2(4\pi)^2p^2}(1 - \frac{m^2}{\Delta}) + \frac{ig_{\mu\nu}}{4(4\pi)^2}(\Gamma(\epsilon) - \Delta_0), \tag{91}$$

$$D = m^2 + p^2(x^2 - x), \Delta_0 = \int_0^1 dx \ln\frac{D}{4\pi\mu^2}, \frac{1}{\Delta} = \int_0^1 \frac{dx}{D}. \tag{92}$$

The integrals I_{a^2b} and $I_{a^2b;\mu}$ are convergent.

Differential equation approach:

$$I_a = -\frac{i}{(4\pi)^2}(m^2(C_0 - 1 + \ln\frac{m^2}{\mu^2}) + C') = I_b, \tag{93}$$

$$I_{a^2} = -\frac{i}{(4\pi)^2}(C_1 + \ln\frac{m^2}{\mu^2}), \tag{94}$$

$$I_{ab} = -\frac{i}{(4\pi)^2}(C_2 + \Delta_0), \tag{95}$$

$$I_{ab;\mu} = -\frac{ip_\mu}{2(4\pi)^2}(C_3 + \Delta_0), \tag{96}$$

$$I_{ab;\mu\nu} = -\frac{ig_{\mu\nu}}{2(4\pi)^2}(2p^2 \int_0^1 dx(x^2 - x)(C_4 - 1 + \ln\frac{D}{\mu^2}) + m^2(C_5 + \Delta_0))$$
$$- \frac{ip_\mu p_\nu}{(4\pi)^2} \int_0^1 dx(1-x)^2(C_4 + \ln\frac{D}{\mu^2}) - \frac{ig_{\mu\nu}p^2}{4(4\pi)^2}C_{4;2} - \frac{ig_{\mu\nu}}{2(4\pi)^2}C_5', \tag{97}$$

$$I_{a^2b} = \frac{-i}{2(4\pi)^2\Delta}, \tag{98}$$

$$I_{a^2b;\mu} = \frac{ip_\mu}{(4\pi)^2p^2}(1 - \frac{m^2}{\Delta}), \tag{99}$$

$$I_{a^2b;\mu\nu} = \frac{ip_\mu p_\nu}{2(4\pi)^2p^2}(1 - \frac{m^2}{\Delta}) - \frac{ig_{\mu\nu}}{4(4\pi)^2}(C_6 + \Delta_0), \tag{100}$$

$$I_{ab^2;\mu\nu} = -\frac{ip_\mu p_\nu}{2(4\pi)^2\Delta} - \frac{3ip_\mu p_\nu}{p^2}(1 - \frac{m^2}{\Delta}) - \frac{ig_{\mu\nu}}{4(4\pi)^2}(C_7 + \Delta_0), \tag{101}$$

$$\Delta_0 = \int_0^1 dx \ln\frac{D}{\mu^2}. \tag{102}$$

In this approach there are 8 dimensionless arbitrary constants (C_0, C_1, C_2, C_3, C_4, C_5, C_6, C_7

and $C_{4;2}$) and three dimensional arbitrary constants (μ, C' and C_5'), all are mass independent. The scale μ here should not be confused with the one in dimensional regularization. However, there are natural relations among these constants that will further reduce the number of independent constants. For example, $I_a^2 = \partial_{m^2} I_a, I_{ab}|_{p=0} = I_{a^2}$, etc. Then we find that $C_0 = C_1 = C_2 = C_3 = C_6 = C_7 = C_5 + 1 = C_4 + C_{4;2}$, hence there are at most five independent constants. Note in employing these relations, metric tensor contraction like $g_{\mu\nu} g^{\nu\mu}$ are not used as such operation is not well defined for the ill-defined integrals, warning example can be found in dimensional schemes.

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